Sparse Spanners vs. Compact Routing

Cyril Gavoille Christian Sommer

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Objective: Great Routing Scheme

- Good Routes
  - Send messages along shortest paths of a network
  - Use paths with low congestion
  - ...
- Efficient Routers
  - Routers store small routing tables only
  - Fast routing decisions
  - ...

Compact Routing on a Graph $G = (V, E)$

- Tradeoff of max Table Size vs. Stretch
- $\text{route}(u, v) \leqslant \alpha \cdot d_G(u, v) + \beta$

Objective: Great Routing Scheme

- Routers store small routing tables

  $\rightsquigarrow$ partial information

  $\rightsquigarrow$ focus on important routes

  $\rightsquigarrow$ edges of a "good" subgraph $\rightsquigarrow$ spanner

Spanners

Spanners have a variety of applications. They are used in space-efficient rout-
ing tables that guarantee almost shortest routes [1], [11], [12], [18], [21], [22],

Subgraphs that

- span the graph (cf. spanning tree)
- maintain certain properties
  here: approximate distances (others: cuts,...)

Stretch

- Graph $G = (V, E)$, Spanner $H = (V, E')$, $E' \subseteq E$
- $d_H(u, v) \leqslant \alpha \cdot d_G(u, v) + \beta$

Spanners

	$\lvert E'\rvert / \lvert V\rvert$	$\alpha\cdot$	$+\beta$
	$O(n)$	1	
[ADD+93]	$\Theta(n^{1/2})$	3	
[ADD+93]	$O(n^{1/k})$	$2k-1$	
[ACIM99]	$\Theta(n^{1/2})$		2
[EP04]	$O(\epsilon^{-1} n^{1/3})$	$1+\epsilon$	4
[BKMP05]	$O(n^{1/3})$		6
[BKMP05]	$O(kn^{1/k})$	k	$k-1$
[Woo06]	$\Omega(n^{1/k}/k)$		$2k-1$

$n := \lvert V\rvert$

$$\leqslant \alpha \cdot d_G(u,v) + \beta$$

Spanners vs. Routing Schemes

	$\lvert E' \rvert \,/\, \lvert V \rvert$	$\alpha \cdot$	$+\beta$	$\lvert$table$\rvert$	
	$O(n)$	1		$O(n)$	
[ADD+93]	$\Theta(n^{1/2})$	3		$O(n^{1/2})$	[TZ01]
[ADD+93]	$\Theta(n^{1/k})$	$O(k)$		$O(n^{1/k})$	[TZ01]
[ACIM99]	$\Theta(n^{1/2})$		2		
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Spanners vs. Routing Schemes

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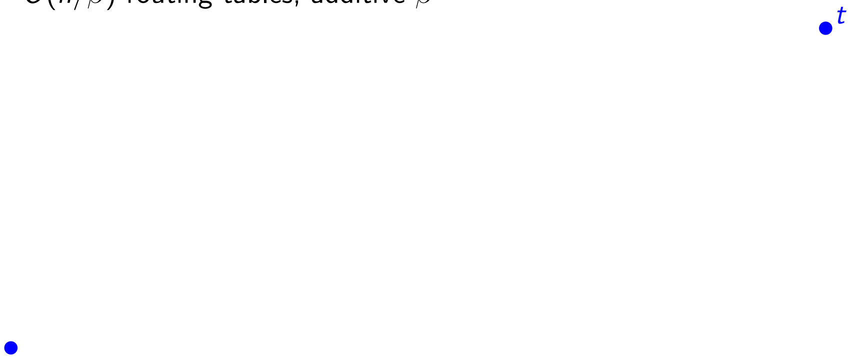
[ACIM99]	$\Theta(n^{1/2})$		2	???	
[EP04]	$O(\epsilon^{-1} n^{1/3})$	$1+\epsilon$	4	???	
[BKMP05]	$O(n^{1/3})$		6	???	
[BKMP05]	$O(kn^{1/k})$	k	$k-1$		
[Woo06]	$\Omega(n^{1/k}/k)$		$2k-1$		
			β	$\tilde{O}(n/\beta)$	Folklore

$n := \lvert V \rvert$

$$\leqslant \alpha \cdot d_G(u,v) + \beta$$

$\tilde{O}(n/\beta)$ routing tables, additive β

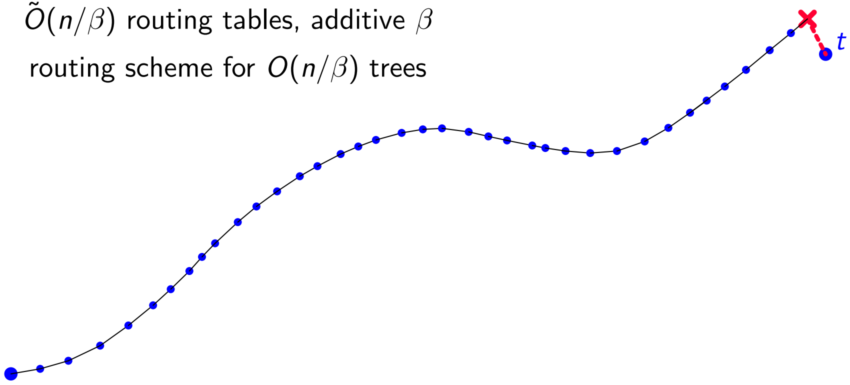


$\tilde{O}(n/\beta)$ routing tables, additive β

$\beta/2$–dominating set (size $O(n/\beta)$)

$\tilde{O}(n/\beta)$ routing tables, additive β

routing scheme for $O(n/\beta)$ trees



Routing Schemes with Additive Stretch

Graph Class	$+\beta$	$\lvert\text{table}\rvert$	
General	β	$\widetilde{O}(n/\beta)$	Folklore
Diameter Δ	2Δ	none	Folklore
$\ell(n)$–Labels	6	$\tilde{O}(\ell(n)\sqrt{n})$	[BC06]
Interval	1	$O(\lg n)$	[BC06]
Circular-arc	1	$O(\lg n)$	[BC06]
Chordal	2	$o(\lg^3 n)$	[DG02]
Tree-length δ	$6\delta - 2$	$O(\delta \lg^2 n)$	[Dou04]
δ-Hyperbolic	$O(\delta \lg n)$	$O(\delta \lg^2 n)$	[CDE+11]

$$\leqslant \alpha \cdot d_G(u,v) + \beta$$

Spanners vs. Routing Schemes

	$\lvert E' \rvert \,/\, \lvert V \rvert$	$\alpha \cdot$	$+\beta$	$\lvert$table$\rvert$	
	$O(n)$	1		$O(n)$	
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[ACIM99]	$\Theta(n^{1/2})$		2	~~???~~ $\tilde{\Omega}(n)$	
[EP04]	$O(\epsilon^{-1} n^{1/3})$	$1+\epsilon$	4		
[BKMP05]	$O(n^{1/3})$		6	~~???~~ $\tilde{\Omega}(n)$	
[BKMP05]	$O(kn^{1/k})$	k	$k-1$		
[Woo06]	$\Omega(n^{1/k}/k)$		$2k-1$	$\tilde{\Omega}(n/k^2)$	
			β	$\tilde{O}(n/\beta)$	Folklore
			β	$\tilde{\Omega}(n/\beta^2)$	NEW

$n := \lvert V \rvert$

$$\leqslant \alpha \cdot d_G(u,v) + \beta$$

Outline

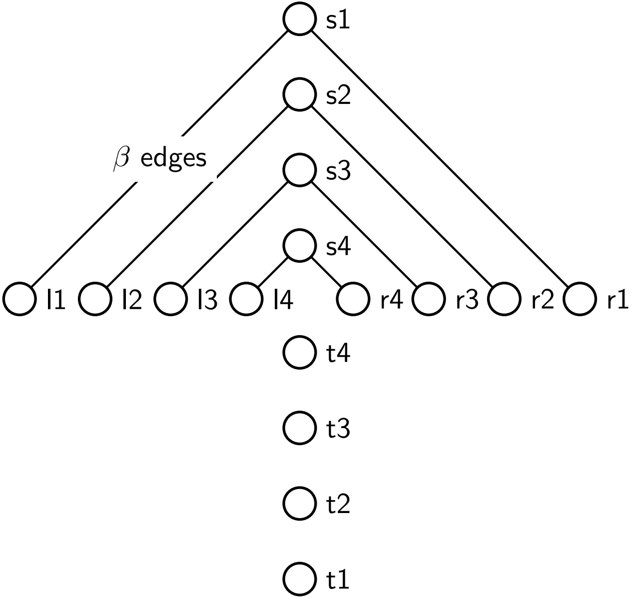
- Motivation and Related Work
- Bad News: Lower Bound
- Good News: Upper Bound
- Conclusion

Lower Bound

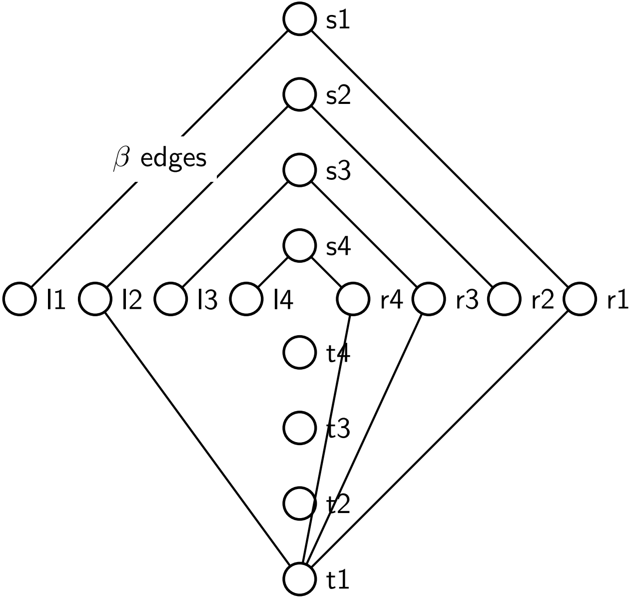
- Assuming polylog addresses
- Additive stretch β implies
- Table size $\tilde{\Omega}(n/\beta^2)$ (planar: $\sqrt{n}/\beta$)

Careful! β–neighborhood can help with additional information

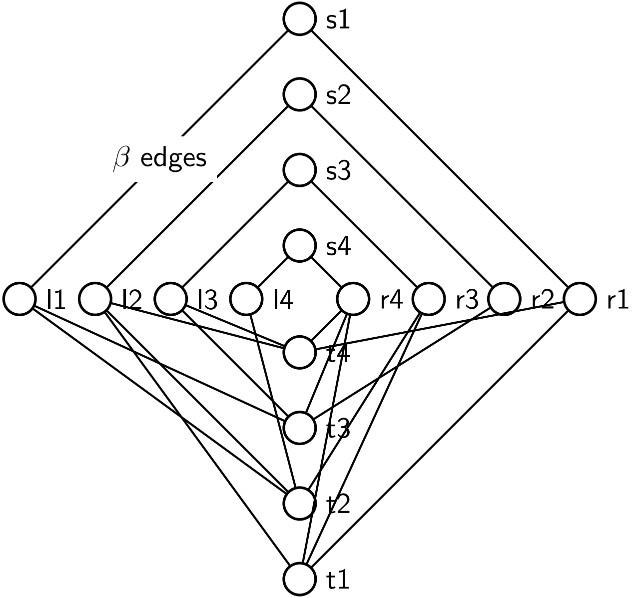
Lower Bound Construction



Lower Bound Construction



Lower Bound Construction



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Upper Bound (for Planar Graphs)

- Additive stretch $\beta = \tilde{O}(n^{1/3})$
- Table size $\tilde{O}(n^{1/3})$

Tight with respect to lower bound for general graphs.

Lower bound for planar graphs: table size $\sqrt{n}/\beta$
(implies $\beta = \tilde{\Omega}(n^{1/2})$ for [Tho04])


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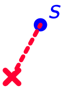
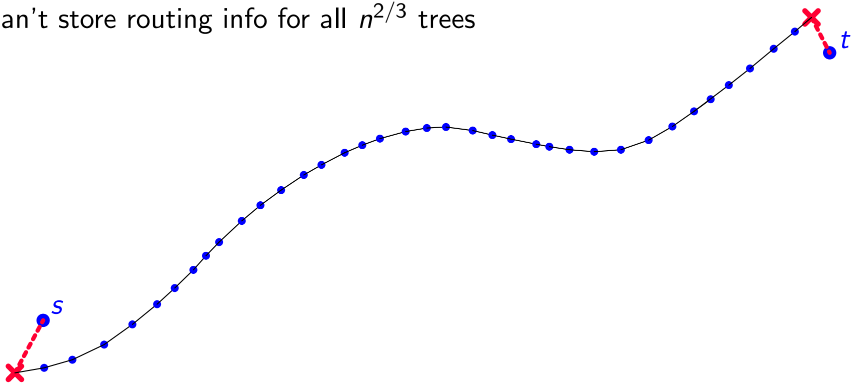
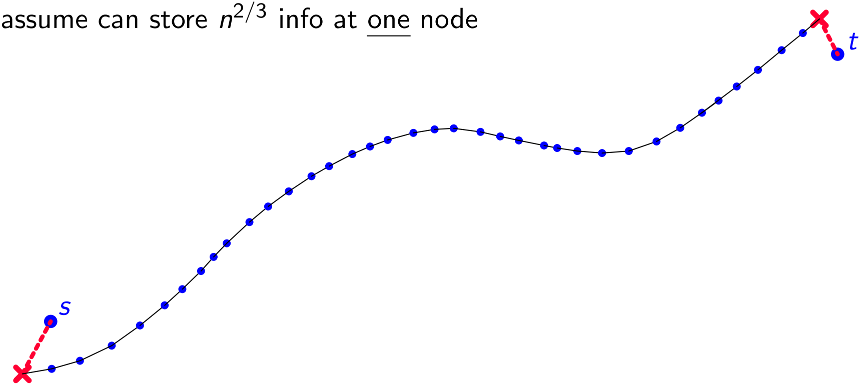
β–dominating set, $\beta = n^{1/3}$



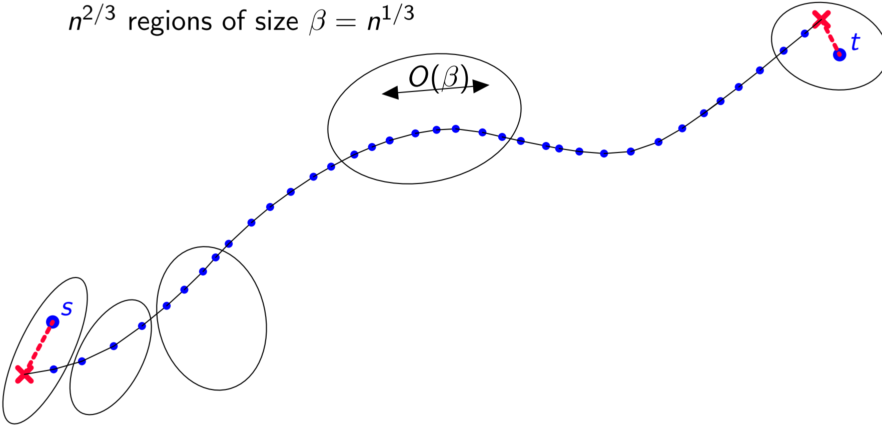
can't store routing info for all $n^{2/3}$ trees



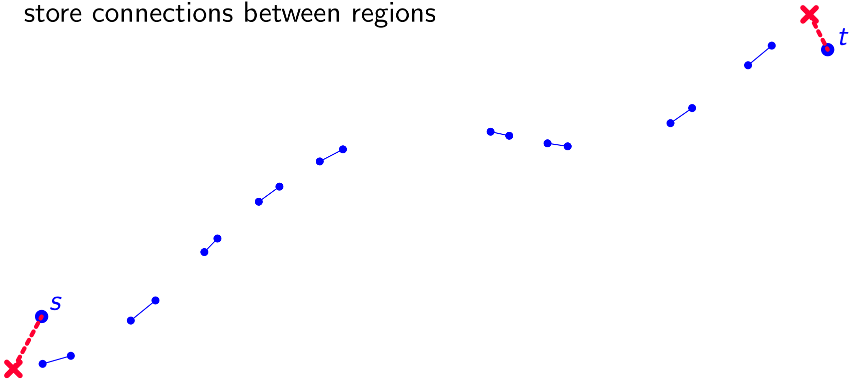
assume can store $n^{2/3}$ info at <u>one</u> node



$n^{2/3}$ regions of size $\beta = n^{1/3}$

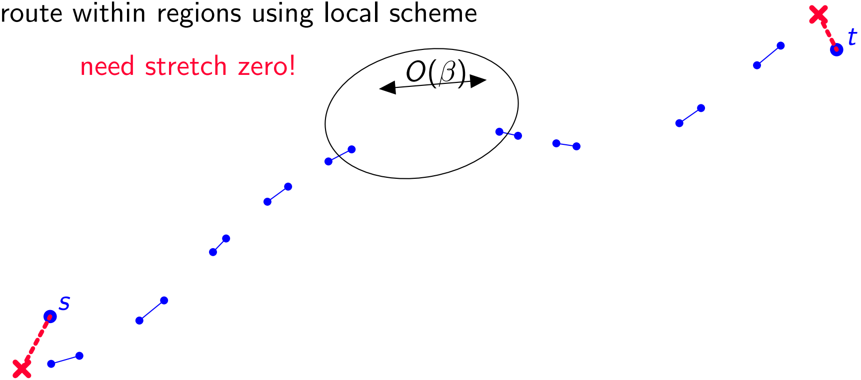


store connections between regions

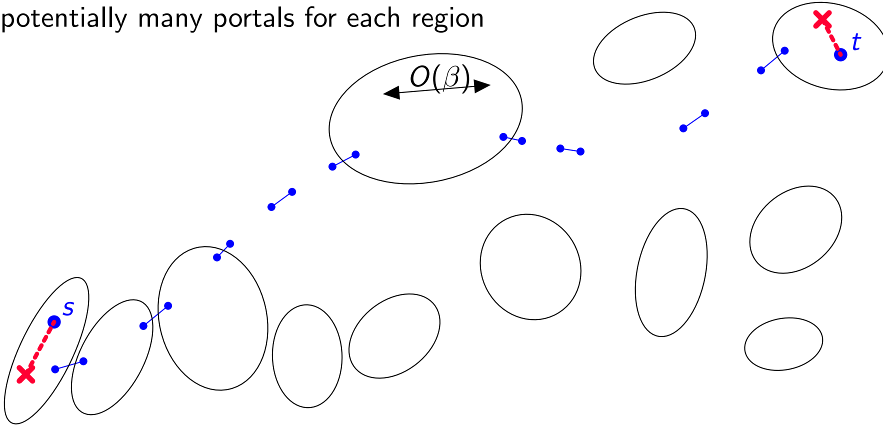


route within regions using local scheme

need stretch zero!

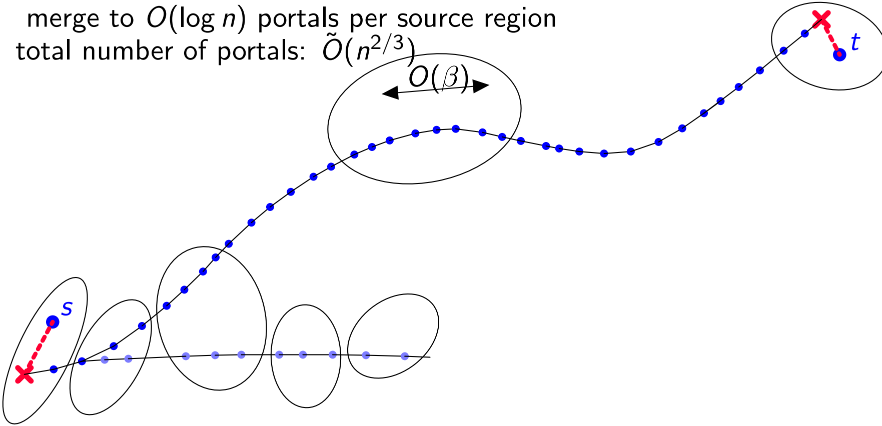


potentially many portals for each region

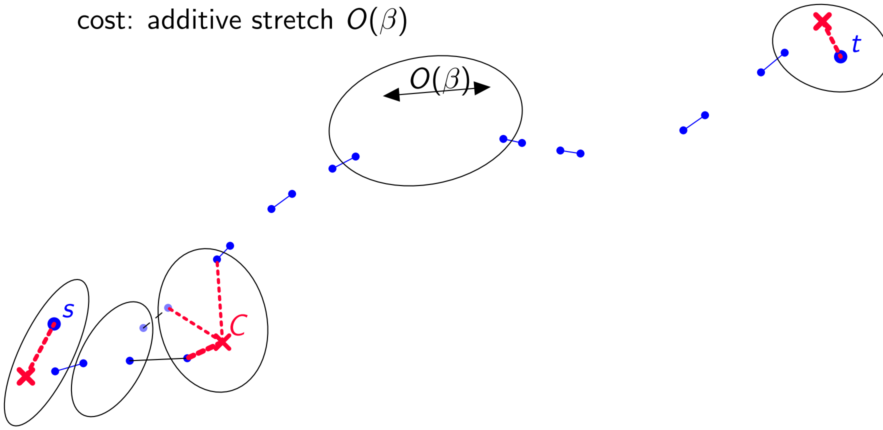


merge to $O(\log n)$ portals per source region

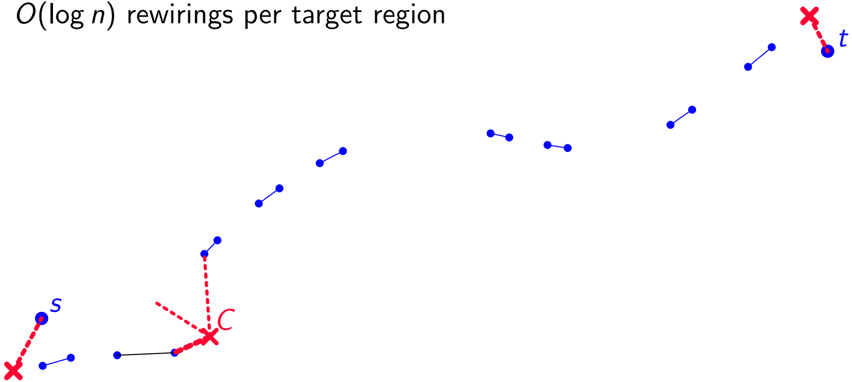
total number of portals: $\tilde{O}(n^{2/3})$



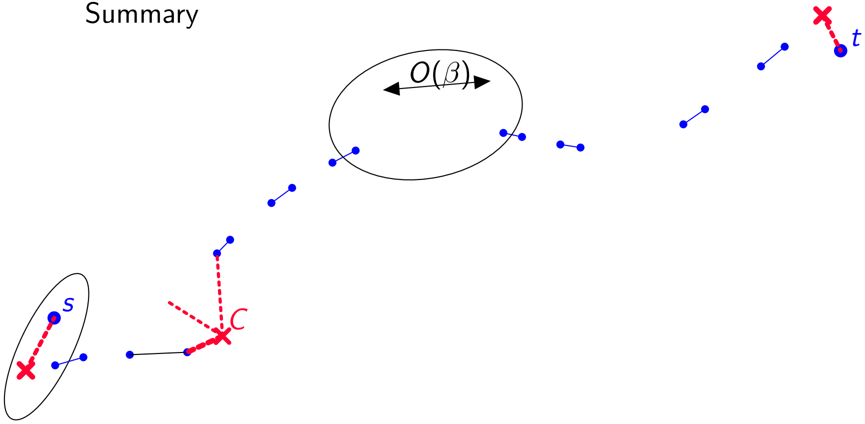
cost: additive stretch $O(\beta)$



$O(\log n)$ rewirings per target region



Summary



Summary

- Table size $n^{1/3}$
  Additive stretch $n^{1/3}$

- for graphs with linear local tree-width
  (needed in local routing scheme)

- Open Problems
  - general graphs, make it tight ;-)
  - reduce header size
  - other points on tradeoff curve

Outline

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Contributions

- Proof that Routing is "harder" than Spanning

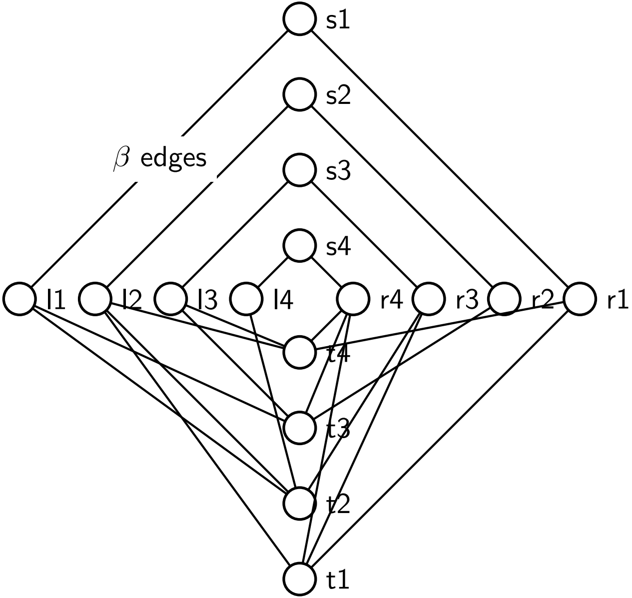
- Lower Bound: stretch β implies table size n/β^2
(trivial upper bound: n/β)

- Upper Bound: routing scheme for planar graphs with table size $n^{1/3}$ and additive stretch $n^{1/3}$

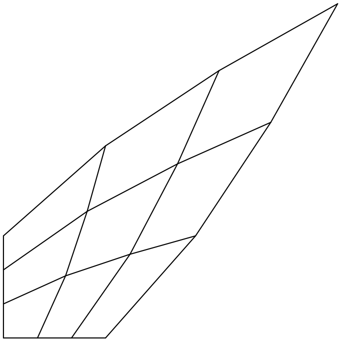
Additional Slides

- Illustration of lower bound for planar graphs

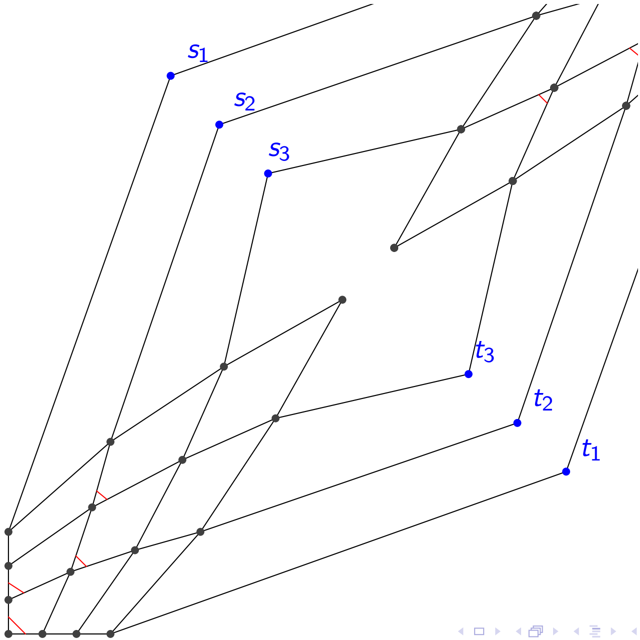
Lower Bound Construction



Lower Bound (Planar) [AGGM06]



Lower Bound (Planar)



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